Teachers' Inquiry in
Mathematics Education

TIMEplate

Template for lesson plans

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Template for lesson plans

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Introduction

This template is a guide on how to do Lesson Study, which means how to study and improve mathematics teaching and students' learning of mathematics.

One-size-fits-all professional development rarely meets the needs of all teachers. Lesson Study differs from more traditional teacher professional development in several ways:

- Focus on the mathematical content, often based on what teachers find difficult to teach, want to improve, or the implementation of a new curriculum.
- Emphasis on an inquiry conducted by peers.
- Collaboration, Lesson Study team members come to feel that lessons are "our" lessons, not "your" or "my" lessons. Members see the contributions of all team teachers and become invested in the colleagues' professional growth.
- Focuses on student learning and development. It provides a valuable chance for teachers to be in a classroom solely to investigate student learning, unencumbered by the need to manage students or provide instructions.

Lesson Study is a collaborative classroom study done by a group of teachers (and researchers). The teachers

- explore teaching practice,
- search for and explore alternatives or innovations,
- conduct studies of students' learning, and
- adjust new approaches.

Lesson Study is an inquiry process that supports teachers to experiment, observe and improve their teaching.

Learning *to do Lesson Study* is the easy part, but it takes time to fully master all processes of the professional development that it involves. Thus, Lesson Study involves continuing efforts to examine and improve teaching approaches: it is not seeking to transmit a single best approach for all teachers. The real "product" of a Lesson Study process is much more than just one polished lesson. As a team collaborates to improve instructions, they deepen their knowledge of mathematical content and student thinking and their commitment to work together to improve instructions. You can find more theoretical information about Lesson Study in *TIMELess: Professional development course on Lesson Study*. The internet offers many resources and a lot of experience on Lesson Study and although those easiest to find concern Lesson Study in the USA or the UK, we should keep in mind that the country, with the most experience and regular practice, is Japan. Adaptation to different countries may offer different opportunities.

The Lesson Study process

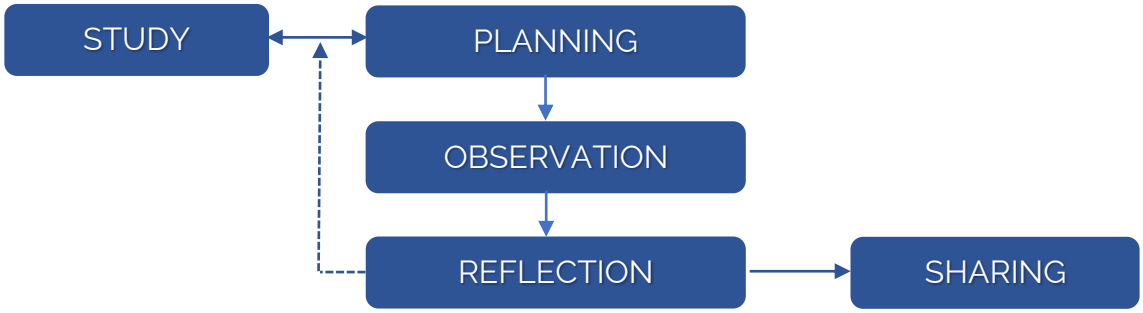

Figure 1. The main phases of Lesson Study, an adaptation of a diagram from Stigler and Hiebert.

Study

The study, selection, and design of tasks in Lesson Study are important. This involvement may be almost invisible for external participants, in the same way that about 90 % of an iceberg is unseen. But exploring the iceberg below the waterline to understand the hidden support that makes it float is comparable to explicate all the mathematical concepts and understand what makes a particular task or problem "float" mathematically.

In the Study phase, the teachers collaborate to study instructional teaching materials, e.g., workbooks and teachers' guidebooks, the literature on the subject matter knowledge, literature on students' difficulties with it. The teachers also draw on earlier experience from their own and others' classroom practices. Teachers may also study the official curriculum and other materials related to the subject matter to be taught in the Study Lesson.

In the Study phase, the teachers can question and investigate the intended mathematical knowledge, e.g., how they can scaffold questioning to help students learn, or how they can enable the students to represent the problem structure in different ways and compare the advantages and disadvantages of different forms of representation. In the process, the deeper knowledge attained will be crucial to design good problems and ways to enable students to learn. It is important to work with an explicit learning goal, before planning the actual teaching in order to reach it.

Planning

A *lesson plan* (see the template, page 8) is a step-by-step guide that provides a structure for the lesson. Planning a lesson may be a long process which ends with drafting the lesson plan. In the lesson plan, the teachers articulate the *target knowledge* and the competences, skills, or other concepts being relevant that are described as *broader goals.* In the plan, it is essential to classify the expected mathematical learning outcomes for the students. A lesson plan helps the teacher in maintaining the focus on students and their work with the topic.

In the lesson plan, it is also important to state the *grade* of the class, the expected *duration,* and the *required materials*, like computer or CAS because it indicates that students are explicitly expected to complete an argument with the materials. If the lesson forms a part of a longer sequence of lessons, this is also explained (overall theme; subthemes of the lessons, including the Study Lesson).

The *problem,* which the students are supposed to work on, must be formulated in such a way that the target knowledge will solve it. To secure this, it is relevant to write down strategies that students might pursue. In the lesson plan the teachers estimate how much time is spent on different phases (devolving the problem, group or individual activities, formulations of hypotheses, validation, and institutionalization of the knowledge developed).

Working on the specific details of how to use the blackboard can help advanced teachers maximize the clarity of the Study Lesson for students. Consider for instance: where to write the problem, where to write (or let the students write) different solutions, etc. The blackboard design can be an appendix to the lesson plan.

Making a lesson plan is a creative process that allows the teachers to elaborate on their knowledge of mathematics they teach, the structure of the wider curriculum, students' thinking, and new ideas for maximizing students' benefits from a lesson.

Variation
Templates for lesson plans occur with some variations. In some adaptations of Lesson Study to cultures outside Japan, there is an explicit focus on selected students, called case students. This seems to make the most sense if all students are well known to the entire team of teachers.

Observation (Study Lesson)

One *selected teacher* is teaching the Study Lesson according to the lesson plan. It is important to agree on Lesson Study as an opportunity to take a risk and try something new, from which everybody will learn. The lesson plan is normally to be followed, but if something unexpected happens, the teacher can divert from the plan, and explain the reasons for this in the post-reflection.

The Lesson Study team, the *Lesson Study guide*, the *External Commentator,* and maybe other invited guests (teachers from other schools, management, etc.) observe and record students' actions. This data collection aims to investigate students' learning and more precisely, the extent to which the goals of the lesson are reached (by some or all students). All observers must have a copy of the lesson plan which describes the goals of the lesson. Observers never interfere with the teaching or communicate with students. Initially, the observers will be placed along the walls of the classroom. During adidactic phases (mainly, students work on the problem) the observers are allowed to move in between students' desks without disturbing, to hear students' discussions and to see what students write. During the Study Lesson, the observers take notes for later discussion, gathering data on how the students respond to the problem, noting how their work is managed by the teacher, and making other notes which are relevant to the goals of the lesson. The main focus should be on the students' action (which cannot be planned but can only be observed).

Reflection

The post-lesson reflection session should, if possible, take place on the same day as the Study Lesson, at best right after, in the same classroom (where materials, blackboard, etc. are still visible). During the reflection session, the teachers and other participants discuss data on students' actions and reflect on the ideas and goals of the lesson, in relation to what was observed. There are some fixed roles or positions for the participants to take. On the panel, in front of the rest of the observers, sit:

- The *Facilitator* is the "host" of the reflection session. He initiates and closes the reflection session. He also assures that the discussion stays on track and ends on time.
- The *Selected teacher* is the teacher who taught the Study Lesson. He is the first to speak after the *Facilitator* initiates the reflection session. He presents reflections, including explanations if and why deviations to the prepared plan occurred during the Study Lesson.
- The *Lesson Study Guide* is the one running and organizing the Lesson Study activities in a Lesson Study team. During the study and preparation of the Study Lesson, he adds comments to help the team of teachers with their learning. In the reflection session, he is the second to speak.

- The *External Commentator* is an experienced Lesson Study participant with special knowledge relevant to the lesson goals. He speaks only at the end of the reflection session and highlights the key learning points from the lesson, not just as a summary of what was said, but more as a reflection on points that had not been adequately made, or wider perspectives that bring wider expertise to the conclusions, as well as open problems to investigate in the future.

Other observers (e.g., other Lesson Study team members) and invited guests are seated in front of the panel. They share their observations after the *Lesson Study Guide*, as the *Facilitator* indicates. The main objective of all comments is to enable the participants to gain explicit, professional knowledge from the lesson and observations. The discussion must be directed (by the *Facilitator*) in such a way that the focus stays on the concrete learning goals of the lesson.

The reflection session thus has four parts (after the Facilitator initiates the reflection session):

I. *The selected teache*r describes her/his view of the lesson. She/he also provides some considerations if she/he had to adjust the teaching compared to the initial plan. Next, the *Lesson Study Guide* recalls the research questions and hypothesis. He describes the lesson plan and its delivery, the reasons behind the lesson plan, and the issues they would like to discuss.
II. Observers (other teachers of the Lesson Study team and invited guests) describe what they saw, based on notes (more rarely: videos or photos). They may also ask questions. The other teachers from the *Lesson Study team* are given priority and may be called on to answer questions.
III. A wider and more open discussion is done by all observers. The *Facilitator* keeps the discussion on track and makes sure the time frame is not exceeded.
IV. The *External Commentator*, who has kept silent until now, highlights some key points to retain from the experience, raising more general, deeper issues and suggesting how participants may pursue these in future lessons.

A list of important things to remember by the Facilitator is:

- Manage the time and content gently but firmly.
- Especially, the subject of the discussion is the observed lesson, not the selected teacher.
- Invite contributions in proper order.
- Ask people to explain themselves.
- Invite people to present "objective" observations.
- Invite people to reflect on how the choices for the lesson plan contributed to the goals being achieved; how these choices worked out in practice.
- Invite people to suggest revisions for the lesson plan.
- Keep the focus on goals – recall these initially.

After the reflection session, the *Lesson Study team* reflect on what they have learned and how they might use it in their teaching. They may also proceed to revise the lesson plan.

Revision of the lesson

In some Lesson Studies, the lesson plan is revised and implemented in another classroom by another (or the same) teacher from the Lesson Study team. In the revision process, the Lesson Study team re-engage in Study, based on the new knowledge from the reflection session. They use this to produce a revised lesson plan. In principle, the process can be repeated several times.

Sharing (Documenting Lesson Study processes)

After the revision process, the lesson plan and the resulting reflections can be shared with other teams and possibly published in educational magazines or presented at conferences. In TIME, teachers are encouraged to make a *Practice report* to support this dissemination. The report is a 2-6 pages long text, written to share the ideas and results developed with other teachers (mainly nationally, but in some cases even further). Of course, there is some difference in what one will emphasise in a report mainly done just for the team involved, and in a report to be published in a journal for teachers. The main object to present in the Practice report is the *Lesson Study team's observations and reflections from practice* (Section "Results" below). The sections of the report could be:

- Title: (usually, the title of the Study Lesson)
- Introduction: Earlier experience or ideas related to the theme.
- Context: Motivation, goals pursued in the Lesson Study, and hypotheses for the present study (based on Study activity).
- Lesson plan: See the Template below.
- Results: Observations from the Study Lesson and main results from the reflection session (can be illustrated with pictures, e.g., of the board or the students' work).
- Conclusion and future perspectives: Based on the results and external comments at the end of the reflection session.
- References: Resources, articles, websites, etc. that were used in the Lesson Study.

The next pages in the TIMEplate are:

- a template of a lesson plan,
- a sample lesson plan (the lesson "Injectivity"),
- a sample practice report (the lesson "Injectivity").

Template of a lesson plan

Target knowledge	
Broader goals	
Prerequisite mathematical knowledge	
Grade	
Time	
Required material	
Problem	

Time	**Teacher's actions incl. instructions**	**Expected students' actions**	**Observations from implementation**
Reflections			

A Lesson Plan for the Study Lesson "Injectivity"

Target knowledge	A formal definition of an injective function
Broader goals	Mathematical communication, identifying injectivity of a function given in various representations, applying injectivity
Prerequisite mathematical knowledge	The concept of function
Grade	10th grade
Time	45 minutes
Required material	Handouts with various representations of functions
Problem	To discover the property of injectivity of a function given in various representations

Time	**Teacher's actions incl. instructions**	**Expected students' actions**	**Observations from implementation**
5 min	The teacher presents the problem: We know linear functions, quadratic functions, polynomials, and we arrange functions by their type. Now we are going to organize functions by some properties.	Students listen.	

	You are given some functions. Find a property which some of the functions have. Sort the functions into two groups – one with those that have the mysterious property and the other one with those that don't have it. Write down arguments for your arrangement.		
10 min	The teacher is observing the students' work.	Students decide how to arrange the given handouts with functions.	
5 min	The teacher asks the groups to place on the right side of the classroom the handouts with the functions having the property and to place the others on the left side.	Students arrange the handouts into two groups.	
5 min	The teacher asks the students to describe how they have decided to arrange the functions.	Students are explaining their reasoning.	
5 min	The teacher asks the groups to switch the functions if they have changed their mind and explain why. If any of the groups can't arrange the functions, the teacher asks all students to suggest necessary changes.	Students are switching the functions.	

10 min	After all the functions have been correctly placed, the teacher declares the property – injectivity. The teacher asks the students to describe the property of injectivity in words.	Students describe the injectivity. They are writing sentences on the board and commenting. They decide which description to accept as the definition.	
5 min	The teacher guides the discussion towards the definition written in mathematical symbols – since injective maps distinct/different elements, what can be said about the arguments if two values of an injective function are the same. The teacher writes the definition of injectivity using formal symbols.	Students answer the questions.	
Reflections			

Practice report of the Study Lesson "Injectivity"

Sanja Antoliš, Matija Bašić, Željka Milin Šipuš, Eva Špalj

Identifying the problem and learning goals

The students of XV. gimnazija mostly continue their education in STEM and therefore it is important that during their high school education, they apply with understanding the mathematical language in defining mathematical terms, formulating mathematical statements, and writing proofs. An insufficient precision in communication as well as reproducing definitions and statements without understanding is a common problem. It is necessary to create a learning environment in which students will first produce an intuitive image of a concept in mathematics and then describe the concept first in spoken and then in as precise mathematical language as possible.

Since the topic of inverse function (exponential and logarithmic function) is highlighted in the curriculum for the second grade of high school, injectivity is chosen. The target knowledge for the Study Lesson is: A formal definition of an injective function, with broader objectives: Mathematical communication, identification of function injectivity in different representations, application of injectivity.

Planning, creating the lesson plan

The lesson is planned for 45 minutes. The material prepared for the lesson includes five sets of functions and two ropes hanging in the classroom. Functions (with finite domains) in each set are the same but given in different representations: formulas, tables, function machines, graphs, and mapping diagrams. The assumption is that different representations will lead to the observation of different aspects of injectivity in the case of finite domains.

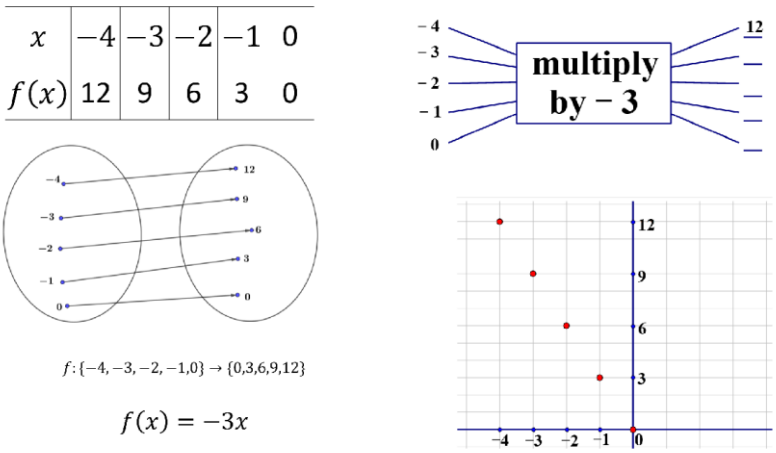

Figure 1. Example of one function given in five different representations.

The students working in groups will be asked to look at their examples and organize the functions into two classes according to a "mysterious property". Students will then describe the property of injectivity and present their observations and descriptions to other groups. Finally, the teacher will give a formal definition of injectivity based on the students' work.

To check whether the goal of the lesson has been achieved, at the end of the lesson each student will give an example of a function that is injective and one that is not injective, using a representation of their choice.

Observation of the Study Lesson

While devolving the task, the teacher emphasized that functions should be considered not according to their type (linear, quadratic, or some other), but according to their properties. After working in groups, only the "green group" with functions presented as mapping diagrams, organized the functions according to injectivity.

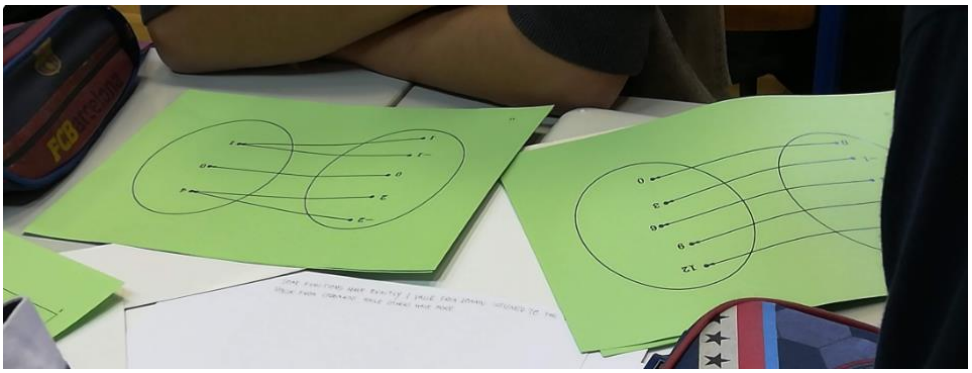

Figure 2. The mapping diagrams of the "green group".

The other groups had different ideas not connected with injectivity and the teacher made a slight change of the plan and led a plenary discussion during which the members of the "green group" revealed the property to others. After that, the students recognized the property in their examples and distributed the papers on two ropes.

Figure 3. The papers hanging on the rope.

The students were asked to write the definition in their own words. Some formulations were written on the board:

- ***A**. Injective functions give different images to different elements.*
- ***B**. Every number will have a different image.*
- ***C**. Each value in the codomain has a unique value in the domain.*
- ***D**. The number of values in the domain and the codomain is always equal.*
- ***E**. Each value in the domain has a unique value in the codomain that no other value in the domain has.*

In the guided discussion, engaging all students, the teacher addressed the imprecisions and misconceptions in some formulations. The discussion did not continue to the symbolic notation of the definition, but that was given to students as a homework assignment.

Reflection and concluding remarks

The selected teacher explained that some decisions to change the lesson plan were made based on the time constraint. In particular, only "the green group" has discovered the property and to save time, the teacher decided to let that group speak in front of the others. It was noticed that it was easier for students to recognize and conceptualize injectivity using the mapping diagrams than in other given representations. Nonetheless, after the idea of the property was specified, the students could solve the classification problem in any representation.

The students were successful in phrasing the definition in words, with certain mistakes. This was expected to some extent, and it showed that the lesson provided an opportunity for the students to exercise using precise mathematical language and that the problem was appropriate for that class.

In the examples at the end of the lesson, some students used the representation they were given at the beginning of the lesson, some used the one in which they understood the property (mapping diagrams), while some used the one, they usually do (formulas).

The reflection phase has finished with the summary of the External Commentator Carl Winsløw from the University of Copenhagen. From the point of view of mathematical communication, it was valuable to see students using different representations, both in an informal and formal language, and expressing themselves orally and in written form. On the other hand, we have seen that the question posed by the teacher did not evoke the desired answer. Students could not check their answers and the teacher was the authority declaring what is right. Nonetheless, it was a situation showing a rich mixture of verbal and non-verbal (even implicit) aspects. From the mathematical point of view, the lesson dealt with the topic that is hard even at the level of university students.