Example of a Lesson Study – Croatia, XV. gimnazija

During November of 2020 our team decided to conduct an online Lesson Study with focus on the domain restriction. Our students' main confusion occurs while solving equations and calculating the square root of a number, they either loose a solution or have two. Similar problem appears in a following school year when they are solving trigonometric functions. We decided to introduce the term *domain restriction* to help them recognize the problem. The Lesson Study was planned for 80 minutes.

The students are given the "set up": a problem of a 5-child family, where each day each sister asks for one piece of fruit. On the other hand, for each day of the week there are different fruits offered but only one of each kind. The goal of the lesson was for fruit to be distributed among children respecting their choosing as much as possible. For example, "choosing" for Wednesday given by the teacher:

"Wednesday: Danijela (banana), Jelena (banana), Maja (pear), Sanja (orange), Vita (plum) **"**

The lesson was conducted in four phases.

First activity – organizing data

Students work in groups to organize given information. Some groups used tables while other used diagrams that we provided.

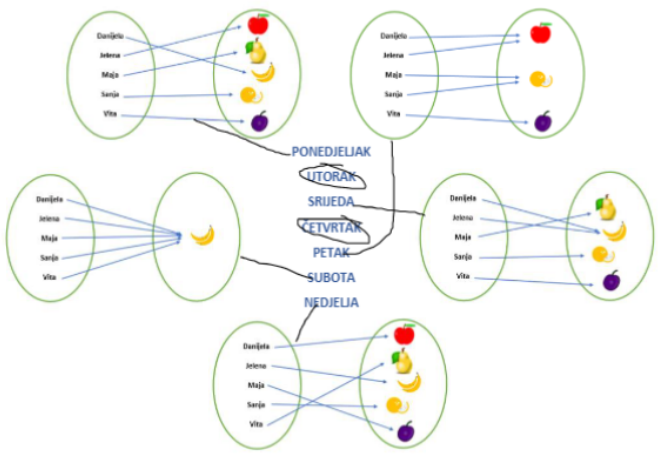


Figure 1: Group 1

Fruit	Danijela	Jelena	Maja	Sanja	Vita	Constrictions	Followed rules?
mon	banana	apple	pear	orange	plum	banana, apple, pear, orange and plum	yes
tue	x	apple	pear	orange	plum	apple, pear, orange and plum	no
wed	banana	banana	pear	orange	plum	banana, pear, orange and plum	yes
thu	banana	apple	pear	orange	plum and pear	banana, pear, orange and plum	no
fri	apple	apple	orange	orange	plum	apple, orange and plum	yes
sat	banana	banana	banana	banana	banana	banana	yes
sun	apple	banana	plum	orange	pear	apple, banana, pear, orange and plum.	yes

green – this person can get what they want

orange – this person can get this if another person who wishes for the same does not

red – this person cannot get what they want

Figure 2: Group 2

Co-funded by the
Erasmus+ Programme
of the European Union

Second activity – choosing and distribution of fruits

Students were asked to draw graphs in the provided coordinate systems. They continued working in groups and had a group discussion about the rules mother proposed (each girl chooses one fruit!)

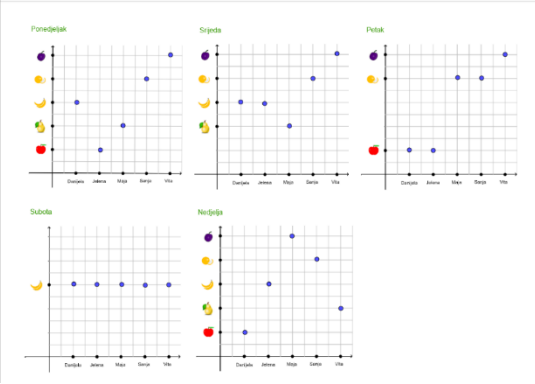


Figure 2: Expected solution from groups 1, 2, 3

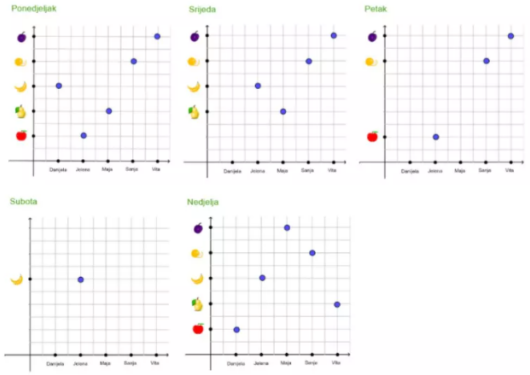


Figure 3: Group 4

Furthermore, group of mothers had to draw „inverse graphs“ that represented “distribution of fruit”. The solutions varied, some students insisted that on some days the problem cannot be solved, some wanted to decide which girl gets the fruit and some of them reached a decision and created a graph.

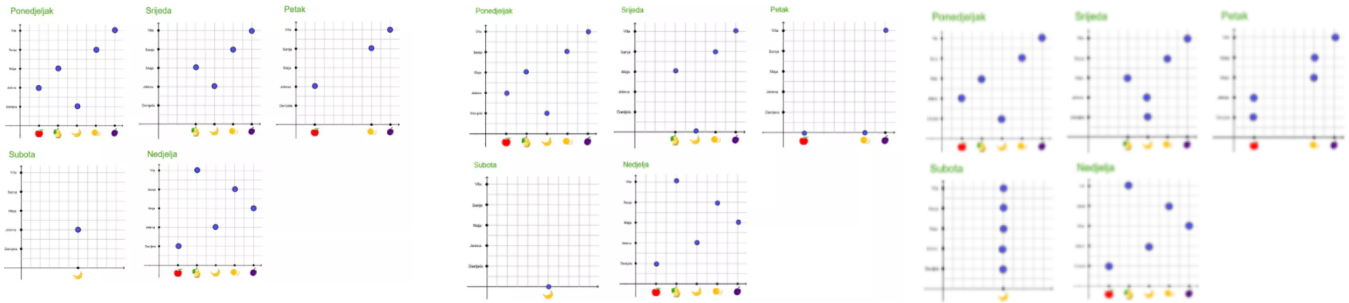


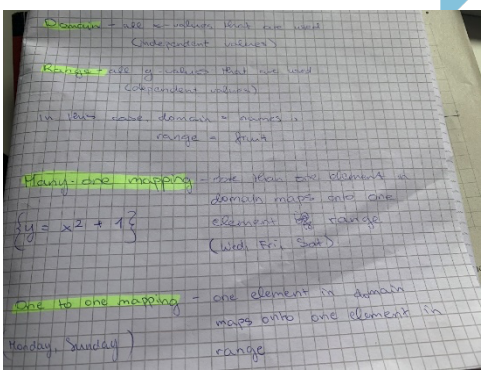
Figure 4: Different solutions from different groups

Third activity – mathematization

Mathematization. Students used the mathematical description of the problem and mostly found the key terms such as- domain, codomain, mapping, function, injection, bijection. Some students realized the necessity of restricting the domain though they have not known the term itself.

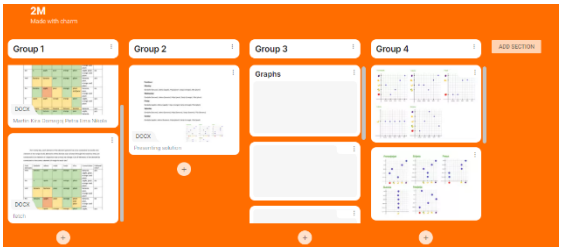
Co-funded by the
Erasmus+ Programme
of the European Union

Try to rewrite this story using as much math terminology as you can.
5 elements in the domain (sisters) have to be mapped with exactly one element in the range (fruit). On Tuesday and Thursday, 2 elements of the domain were either connected with 2 or no elements in the range. For this reason, they were not functions. Other 5 days were following the rule and were functions. However, on Wednesday, Friday and Satruday there are multiple elements form the domain connected to the same element in the range. This works for it being a function, yet there is a restriction of fruit available, meaning that not all girl can get a whole fruit. Perhaps they can share it? Or fight for it?



Fourth activity – presentations and institutionalization

Each group presented their work by posting on Padlet and then we were able to revise the known mathematical terms considering functions. The teacher introduced and defined the new term – domain restriction.



The lesson was implemented four times by different teachers also used as an introduction to functions in general.

The feedback from students was somewhat limited because of the online environment but the students were given an online survey where they expressed their opinion and gave suggestions on how to improve the lesson. Mainly, they liked the idea of investigating but also felt that they needed more guidance from the teacher.

The lesson was also discussed between the team members and some interesting questions have arisen, such as: “How the lesson could be adapted for in person teaching instead of online?”, “Could the tasks be less structured?” etc. We, as a team look forward to answering all those questions, and others, hopefully very soon.

Co-funded by the
Erasmus+ Programme
of the European Union